About lies and other statistics

How to use Z tables for the normal distribution?

"There are three types of lies - lies, damn lies, and statistics." (Benjamin Disraeli)

HOW TO FIND PROBABILITIES FOR A NORMAL DISTRIBUTION USING Z TABLES?

Assume a variable x that follows a normal distribution. As you know, a normal distribution is characterised by an value for the average μ, and a standard deviation σ, further abbreviated as $N(\mu,\sigma)$.

Any normal distribution $N(\mu,\sigma)$ can be transformed to a standard normal distribution N(0,1) as follows:

$$z = (x - \mu) / \sigma$$

This z-value corresponds to the x-value: the x-value follows a $N(\mu,\sigma)$ just as the z-value follows a N(0,1). So this means that whether you give answers for x or for z, the answers are the same!

But here is the good news: You can use standardised normal tables or Z tables and answers questions for z-values.

Sounds unbelievable? It's the truth! You can indeed use the Z table to find a full set of "less-than" or "more than" probabilities for a wide range of z-values. To use the Z table to find probabilities for a statistical sample with a standard normal (Z-) distribution, do the following:

Step 1. Go to the row that represents the ones digit and the first digit after the decimal point (the tenths digit) of your z-value.

Step 2. Go to the column that represents the second digit after the decimal point (the hundredths digit) of your z-value.

Step 3. Intersect the row and column from Steps 1 and 2. This result represents $p(Z < z)$, the probability that the random variable Z is less than the value z (also known as the percentage of z-values that are less than the given z value).

Example. Suppose you have a variable X following N(5.5;1.5), and you want to calculate the probability that a value is smaller than x = 8.695, i.e. $P(X \leq 8.695)$.

This is exactly the same question as $P(Z \leq 2.13)$, since $z = (8.695 - 5.5) / 1.5 = 2.13$.

So you want to find $p(Z \leq 2.13)$. Using the Z table below, find the row for 2.1 and the column for 0.03. Intersect that row and column to find the probability: 0.9834. Therefore $p(Z \leq 2.13) = 0.9834$ or slightly more than 98%!

Remark. There is no difference between using ≤ (smaller than or equal to) and < (smaller than) so that means that $P(Z \leq 2.13)$ is exactly the same as $P(Z < 2.13)$.

Noting that the total area under any normal curve (including the standardised normal curve) is 1, it follows that $p(Z < 2.13) + p(Z > 2.13) = 1$. Therefore, $p(Z > 2.13) = 1 - p(Z < 2.13)$ which equals 1 – 0.9834 which equals 0.0166.

Suppose you want to look for $p(Z < -2.13)$. You find the row for –2.1 and the column for 0.03. Intersect the row and column and you find 0.0166; that means $p(Z < -2.13) = 0.0166$. Observe that this happens to equal $p(Z > +2.13)$. The reason for this is because the normal distribution is symmetric. So the tail of the curve below –2.13 representing $p(Z < -2.13)$ looks exactly like the tail above 2.13 representing $p(Z > +2.13)$. Magnificent, isn't it?

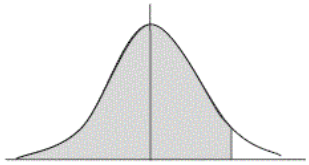
You can do much more with these tables! Do you want to learn how?

1. Practice with them a little bit
2. Take a look at YouTube (e.g. https://www.youtube.com/watch?v=IgwT6tDniko)
3. Use a function in MS Excel (=NORM.DIST(8.695,5.5,1.5,1)) and you'll have the same answer.

Z TABLE FOR THE STANDARDISED NORMAL DISTRIBUTION

A table for the standardised normal distribution can be found anywhere (Google it!) but below you find one that is easy to use.

Standard Normal Distribution



$$p(z \le z_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_1} e^{-\frac{1}{2}z^2} dz$$

z_1	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000